# Computer-Supported Program Verification with the RISC ProofNavigator

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1. An Overview of the RISC ProofNavigator

2. Specifying Arrays

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# The RISC ProofNavigator

- An interactive proving assistant for program verification.
  - Research Institute for Symbolic Computation (RISC), 2005–:
    http://www.risc.uni-linz.ac.at/
    research/formal/software/ProofNavigator.
  - Development based on prior experience with PVS (SRI, 1993–).
  - Kernel and GUI implemented in Java.
  - Uses external SMT (satisfiability modulo theories) solver.
    - CVCL (Cooperating Validity Checker Lite) 2.0.
  - Runs under Linux (only); freely available as open source (GPL).
- A language for the definition of logical theories.
  - Based on a strongly typed higher-order logic (with subtypes).
  - Introduction of types, constants, functions, predicates.
- Computer support for the construction of proofs.
  - Commands for basic inference rules and combinations of such rules.
  - Applied interactively within a sequent calculus framework.
  - Top-down elaboration of proof trees.

Designed for simplicity of use; applied to non-trivial verifications.

# Using the Software

For survey, see "Program Verification with the RISC ProofNavigator".
For details, see "The RISC ProofNavigator: Tutorial and Manual".

- Develop a theory.
  - Text file with declarations of types, constants, functions, predicates.
  - Axioms (propositions assumed true) and formulas (to be proved).
- Load the theory.
  - File is read; declarations are parsed and type-checked.
  - Type-checking conditions are generated and proved.
- Prove the formulas in the theory.
  - Human-guided top-down elaboration of proof tree.
  - Steps are recorded for later replay of proof.
  - Proof status is recorded as "open" or "completed".
- Modify theory and repeat above steps.
  - Software maintains dependencies of declarations and proofs.
  - Proofs whose dependencies have changed are tagged as "untrusted".

# Starting the Software

- Starting the software:

```
ProofNavigator &      (32 bit machines at RISC)
ProofNavigator64 &    (64 bit machines at RISC)
```

- Command line options:

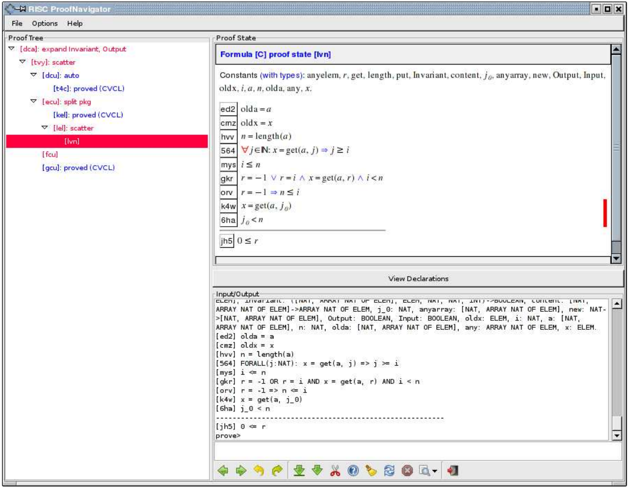
```
Usage: ProofNavigator [OPTION]... [FILE]
FILE:   name of file to be read on startup.
OPTION: one of the following options:
        -n, --nogui:  use command line interface.
        -c, --context NAME: use subdir NAME to store context.
        --cvcl PATH: PATH refers to executable "cvcl".
        -s, --silent: omit startup message.
        -h, --help:   print this message.
```

- Repository stored in subdirectory of current working directory:

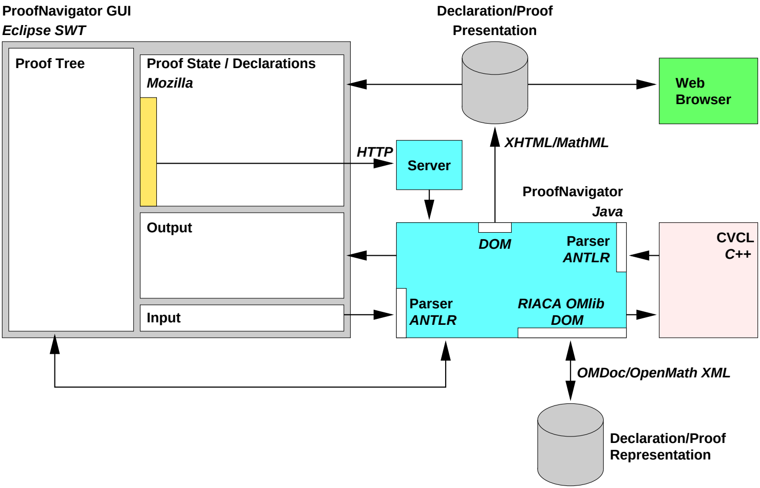
  `.ProofNavigator/`

  - Option `-c` *dir* or command `newcontext "`*dir*`"`:
    - Switches to repository in directory *dir*.

# The Graphical User Interface



# The Software Architecture



# Software Components

- Graphical user interface.
  - Display of declarations and proof state.
  - Embeds HTML browser as core component.
- Proof engine.
  - Commands for navigating the proof.
  - Interaction with validity checker to simplify/close proof states.
- Validity checker.
  - Simplifies formulas
  - Checks the validity of formulas.
  - Produces counterexamples for (presumedly) invalid formulas.
- Object repository.
  - Proof persistence.
  - Proof status management.

All data are externally represented in (gzipped) XML.

# A Theory

```
% switch repository to "sum"
newcontext "sum";

% the recursive definition of the sum from 0 to n
sum: NAT->NAT;
S1: AXIOM sum(0)=0;
S2: AXIOM FORALL(n:NAT): n>0 => sum(n)=n+sum(n-1);

% proof that explicit form is equivalent to recursive definition
S: FORMULA FORALL(n:NAT): sum(n) = (n+1)*n/2;
```

Declarations written with an external editor in a text file.

# Proving a Formula

When the file is loaded, the declarations are pretty-printed:

$$\mathrm{sum} \in \mathbb{N} \to \mathbb{N}$$

$$\textbf{axiom}\ \mathrm{S1} \equiv \mathrm{sum}(0) = 0$$

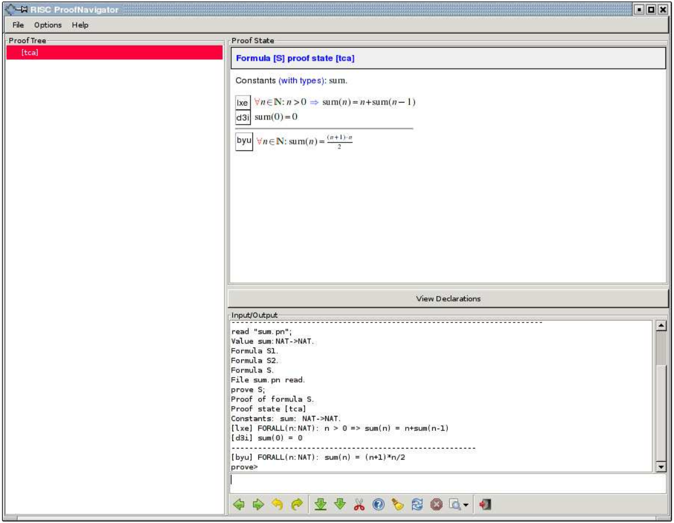
$$\textbf{axiom}\ \mathrm{S2} \equiv \forall n \in \mathbb{N}: n > 0 \Rightarrow \mathrm{sum}(n) = n + \mathrm{sum}(n-1)$$

$$S \equiv \forall n \in \mathbb{N}: \mathrm{sum}(n) = \frac{(n+1)\cdot n}{2}$$

The proof of a formula is started by the `prove` command.

| Formula S |
|---|
| prove S: Construct Proof |
| proof S: Show Proof |
| formula S: Print Formula |

# Proving a Formula



# Proving a Formula

- Proof of formula $F$ is represented as a tree.
  - Each tree node denotes a proof state (*goal*).
    - Logical sequent:
      $A_1, A_2, \ldots \vdash B_1, B_2, \ldots$.
    - Interpretation:
      $(A_1 \wedge A_2 \wedge \ldots) \Rightarrow (B_1 \vee B_2 \vee \ldots)$
  - Initially single node $Axioms \vdash F$.

$$
\begin{array}{ll}
\multicolumn{2}{l}{\text{Constants: } x_0 \in S_0, \ldots} \\
[L_1] & A_1 \\
\ldots & \\
[L_n] & A_n \\
\hline
[L_{n+1}] & B_1 \\
\ldots & \\
[L_{n+m}] & B_m
\end{array}
$$

- The tree must be expanded to completion.
  - Every leaf must denote an obviously valid formula.
    - Some $A_i$ is false or some $B_j$ is true.
- A proof step consists of the application of a proving rule to a goal.
  - Either the goal is recognized as true.
  - Or the goal becomes the parent of a number of children (subgoals).

    The conjunction of the subgoals implies the parent goal.

# An Open Proof Tree

Proof Tree

- [tca]: induction n in byu
  - [dbj]: proved (CVCL)
  - [ebj]

**Formula [S] proof state [dbj]**

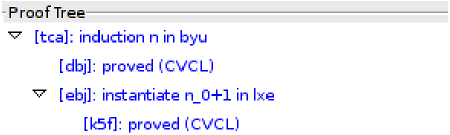
Constants (with types): sum.

| | |
|---|---|
| lxe | $\forall n \in \mathbb{N}: n > 0 \Rightarrow \mathrm{sum}(n) = n + \mathrm{sum}(n-1)$ |
| d3i | $\mathrm{sum}(0) = 0$ |
| nfq | $\mathrm{sum}(0) = \frac{(0+1)\cdot 0}{2}$ |

**Parent: [tca]**

Closed goals are indicated in blue; goals that are open (or have open subgoals) are indicated in red. The red bar denotes the “current” goal.

# A Completed Proof Tree




The visual representation of the complete proof structure; by clicking on a node, the corresponding proof state is displayed.

# Navigation Commands

Various buttons support navigation in a proof tree.

- : `prev`
  - Go to previous open state in proof tree.
- : `next`
  - Go to next open state in proof tree.
- : `undo`
  - Undo the proof command that was issued in the parent of the current state; this discards the whole proof tree rooted in the parent.
- : `redo`
  - Redo the proof command that was previously issued in the current state but later undone; this restores the discarded proof tree.

Single click on a node in the proof tree displays the corresponding state; double click makes this state the current one.

# Proving Commands

The most important proving commands can be also triggered by buttons.

- (scatter)
  - Recursively applies decomposition rules to the current proof state and to all generated child states; attempts to close the generated states by the application of a validity checker.
- (decompose)
  - Like `scatter` but generates a single child state only (no branching).
- (split)
  - Splits current state into multiple children states by applying rule to current goal formula (or a selected formula).
- (auto)
  - Attempts to close current state by instantiation of quantified formulas.
- (autostar)
  - Attempts to close current state and its siblings by instantiation.

Automatic decomposition of proofs and closing of proof states.

# Proving Commands

More commands can be selected from the menus.

- `assume`
  - Introduce a new assumption in the current state; generates a sibling state where this assumption has to be proved.
- `case`:
  - Split current state by a formula which is assumed as true in one child state and as false in the other.
- `expand`:
  - Expand the definitions of denoted constants, functions, or predicates.
- `lemma`:
  - Introduce another (previously proved) formula as new knowledge.
- `instantiate`:
  - Instantiate a universal assumption or an existential goal.
- `induction`:
  - Start an induction proof on a goal formula that is universally quantified over the natural numbers.

Here the creativity of the user is required!

# Auxiliary Commands

Some buttons have no command counterparts.

- : `counterexample`
  - Generate a “counterexample” for the current proof state, i.e. an interpretation of the constants that refutes the current goal.
- 
  - Abort current prover activity (proof state simplification or counterexample generation).
- 
  - Show menu that lists all commands and their (optional) arguments.

More facilities for proof control.

# Proving Strategies

- Initially: semi-automatic proof decomposition.
  - `expand` expands constant, function, and predicate definitions.
  - `scatter` aggressively decomposes a proof into subproofs.
  - `decompose` simplifies a proof state without branching.
  - `induction` for proofs over the natural numbers.
- Later: critical hints given by user.
  - `assume` and `case` cut proof states by conditions.
  - `instantiate` provide specific formula instantiations.
- Finally: simple proof states are yielded that can be automatically closed by the validity checker.
  - `auto` and `autostar` may help to close formulas by the heuristic instantiation of quantified formulas.

Appropriate combination of semi-automatic proof decomposition, critical hints given by the user, and the application of a validity checker is crucial.

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# A Constructive Definition of Arrays

```
% constructive array definition
newcontext "arrays2";

% the types
INDEX: TYPE = NAT;
ELEM:  TYPE;
ARR:   TYPE =
  [INDEX, ARRAY INDEX OF ELEM];

% error constants
any:      ARRAY INDEX OF ELEM;
anyelem:  ELEM;
anyarray: ARR;

% a selector operation
content:
  ARR -> (ARRAY INDEX OF ELEM) =
  LAMBDA(a:ARR): a.1;
```

```
% the array operations
length: ARR -> INDEX =
  LAMBDA(a:ARR): a.0;
new: INDEX -> ARR =
  LAMBDA(n:INDEX): (n, any);
put: (ARR, INDEX, ELEM) -> ARR =
 LAMBDA(a:ARR, i:INDEX, e:ELEM):
   IF i < length(a)
     THEN (length(a),
           content(a) WITH [i]:=e)
     ELSE anyarray
   ENDIF;
get: (ARR, INDEX) -> ELEM =
  LAMBDA(a:ARR, i:INDEX):
    IF i < length(a)
      THEN content(a)[i]
      ELSE anyelem ENDIF;
```

# Proof of Fundamental Array Properties

```
% the classical array axioms as formulas to be proved
length1: FORMULA
  FORALL(n:INDEX): length(new(n)) = n;

length2: FORMULA
  FORALL(a:ARR, i:INDEX, e:ELEM):
    i < length(a) => length(put(a, i, e)) = length(a);

get1: FORMULA
  FORALL(a:ARR, i:INDEX, e:ELEM):
    i < length(a) => get(put(a, i, e), i) = e;

get2: FORMULA
  FORALL(a:ARR, i, j:INDEX, e:ELEM):
    i < length(a) AND j < length(a) AND
    i /= j =>
      get(put(a, i, e), j) = get(a, j);
```

[adu]: expand length, get, put, content
  [c3b]: scatter
    [qid]: proved (CVCL)

# Proof of a Higher-Level Array Property

```
% extensionality on low-level arrays
extensionality: AXIOM
  FORALL(a, b:ARRAY INDEX OF ELEM):
    a=b <=> (FORALL(i:INDEX):a[i]=b[i]);

% unassigned parts hold identical values
unassigned: AXIOM
  FORALL(a:ARR, i:INT):
    (i >= length(a)) => content(a)[i

% extensionality on arrays to be prc
equality: FORMULA
  FORALL(a:ARR, b:ARR): a = b <=>
    length(a) = length(b) AND
    (FORALL(i:INDEX): i < length(a) => get(a,i) = get(b,i));
```

[adt]: expand length, get, content
  [cw2]: scatter
    [qey]: proved (CVCL)
    [rey]: assume b_0.1 = a_0.1
      [zpt]: proved (CVCL)
      [1pt]: instantiate a_0.1, b_0.1 in 1fm
        [y51]: scatter
          [ku2]: auto
            [iub]: proved (CVCL)

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# A Program Verification

Verification of the following Hoare triple:

$$\{olda = a \wedge oldx = x \wedge n = |a| \wedge i = 0 \wedge r = -1\}$$

**while** $i < n \wedge r = -1$ **do**
  **if** $a[i] = x$
    **then** $r := i$
    **else** $i := i + 1$

$$\{a = olda \wedge x = oldx \wedge$$
$$((r = -1 \wedge \forall i : 0 \le i < |a| \Rightarrow a[i] \ne x) \vee$$
$$(0 \le r < |a| \wedge a[r] = x \wedge \forall i : 0 \le i < r \Rightarrow a[i] \ne x))\}$$

Find the smallest index $r$ of an occurrence of value $x$ in array $a$ ($r = -1$, if $x$ does not occur in $a$).

# The Verification Conditions

$A :\Leftrightarrow \mathit{Input} \Rightarrow \mathit{Invariant}$
$B_1 :\Leftrightarrow \mathit{Invariant} \wedge i < n \wedge r = -1 \wedge a[i] = x \Rightarrow \mathit{Invariant}[i/r]$
$B_2 :\Leftrightarrow \mathit{Invariant} \wedge i < n \wedge r = -1 \wedge a[i] \neq x \Rightarrow \mathit{Invariant}[i+1/i]$
$C :\Leftrightarrow \mathit{Invariant} \wedge \neg(i < n \wedge r = -1) \Rightarrow \mathit{Output}$

$\mathit{Input} :\Leftrightarrow \mathit{olda} = a \wedge \mathit{oldx} = x \wedge n = \mathit{length}(a) \wedge i = 0 \wedge r = -1$

$\mathit{Output} :\Leftrightarrow a = \mathit{olda} \wedge x = \mathit{oldx} \wedge$
$((r = -1 \wedge \forall i : 0 \leq i < \mathit{length}(a) \Rightarrow a[i] \neq x) \vee$
$(0 \leq r < \mathit{length}(a) \wedge a[r] = x \wedge \forall i : 0 \leq i < r \Rightarrow a[i] \neq x))$

$\mathit{Invariant} :\Leftrightarrow \mathit{olda} = a \wedge \mathit{oldx} = x \wedge n = \mathit{length}(a) \wedge$
$0 \leq i \leq n \wedge \forall j : 0 \leq j < i \Rightarrow a[j] \neq x \wedge$
$(r = -1 \vee (r = i \wedge i < n \wedge a[r] = x))$

The verification conditions $A, B_1, B_2, C$ have to be proved.

# The Verification Conditions

```
newcontext
  "linsearch";

% declaration
% of arrays
...

a: ARR;
olda: ARR;
x: ELEM;
oldx: ELEM;
i: NAT;
n: NAT;
r: INT;
```

```
Input: BOOLEAN = olda = a AND oldx = x AND
  n = length(a) AND i = 0 AND r = -1;

Output: BOOLEAN = a = olda AND
  ((r = -1 AND
     (FORALL(j:NAT): j < length(a) =>
        get(a,j) /= x)) OR
   (0 <= r AND r < length(a) AND get(a,r) = x AND
     (FORALL(j:NAT):
       j < r => get(a,j) /= x)));

Invariant: (ARR, ELEM, NAT, NAT, INT) -> BOOLEAN =
  LAMBDA(a: ARR, x: ELEM, i: NAT, n: NAT, r: INT):
    olda = a AND oldx = x AND
    n = length(a) AND i <= n AND
    (FORALL(j:NAT): j < i => get(a,j) /= x) AND
    (r = -1 OR (r = i AND i < n AND get(a,r) = x));
...
```

# The Verification Conditions (Contd)

```
...

A: FORMULA
  Input => Invariant(a, x, i, n, r);

B1: FORMULA
  Invariant(a, x, i, n, r) AND i < n AND r = -1 AND get(a,i) = x
    => Invariant(a,x,i,n,i);

B2: FORMULA
  Invariant(a, x, i, n, r) AND i < n AND r = -1 AND get(a,i) /= x
    => Invariant(a,x,i+1,n,r);

C: FORMULA
  Invariant(a, x, i, n, r) AND NOT(i < n AND r = -1)
    => Output;
```

# The Proofs

A:

- [bca]: expand Input, Invariant
  - [fuo]: scatter
    - [bxg]: proved (CVCL)

(2 user actions)

B1:

- [p1b]: expand Invariant
  - [lf6]: proved (CVCL)

(1 user action)

B2:

- [q1b]: expand Invariant in 6kv
  - [slx]: scatter
    - [a1y]: auto
      - [cch]: proved (CVCL)
    - [b1y]: proved (CVCL)
    - [c1y]: proved (CVCL)
    - [d1y]: proved (CVCL)
    - [e1y]: proved (CVCL)

(3 user actions)

C:

- [dca]: expand Invariant, Output in zfg
  - [tvy]: scatter
    - [dcu]: auto
      - [t4c]: proved (CVCL)
    - [ecu]: split pkg
      - [kel]: proved (CVCL)
      - [lel]: scatter
        - [lvn]: auto
          - [lap]: proved (CVCL)
    - [fcu]: auto
      - [blt]: proved (CVCL)
    - [gcu]: proved (CVCL)

(6 user actions)

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# Conclusions

So what does this experience show us?

- Parts of a verification can be handled quite automatically:
  - Top-down proof decomposition.
  - Propositional logic reasoning.
  - Equality reasoning.
  - Linear arithmetic.
- Manual control for crucial “creative steps”
  - Expansion of definitions.
  - Proof cuts by assumptions/case distinctions.
  - Application of additional lemmas.
  - Instantiation of quantified formulas.

Proving assistants can do the essentially simple but usually tedious parts of the proof; the human nevertheless has to provide the creative insight.

# Popular Proving Assistants

- PVS: `http://pvs.csl.sri.com`
  - SRI (Software Research Institute) International, Menlo Park, CA.
  - Integrated environment for developing and analyzing formal specs.
  - Core system is implemented in Common Lisp.
  - Emacs-based frontend with Tcl/Tk-based GUI extensions.
- Isabelle/HOL: `http://isabelle.in.tum.de`
  - University of Cambridge and Technical University Munich.
  - Isabelle: generic theorem proving environment (aka "proof assistant").
  - Isabelle/HOL: instance that uses higher order logic as framework.
  - Decisions procedures, tactics for interactive proof development.
- Coq: `http://coq.inria.fr`
  - LogiCal project, INRIA, France.
  - Formal proof management system (aka "proof assistant").
  - "Calculus of inductive constructions" as logical framework.
  - Decision procedures, tactics support for interactive proof development.